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Over the past several years with support from AFOSR, we have been engaged in the development of theory and algorithms for control and estimation of highly nonlinear systems. The research has focused on several areas, nonlinear H-infinity control, nonlinear detectability and the solution of PDE's that arise in nonlinear control. We have developed necessary and sufficient conditions for nonlinear H-infinity control, necessary conditions for nonlinear detectability, a general theory of nonlinear observers and software for the term solution of some of the PDE's arising in nonlinear control and estimation.

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Final Technical Report
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Abstract

Over the past several years with support from AFOSR, we have been engaged in the development of theory and algorithms for control and estimation of highly nonlinear systems. The research has focused on several areas, nonlinear H_∞ control, nonlinear detectability and the solution of PDE's that arise in nonlinear control. We have developed necessary and sufficient conditions for nonlinear H_∞ nontrol, necessary conditions for nonlinear detectability, a general theory of nonlinear observers and software for the term by term solution of some of the PDE's arising in nonlinear control and estimation.

1 Introduction

Over the past several years with support from AFOSR, we have been engaged in the development of theory and algorithms for control and estimation of highly nonlinear systems such as high performance aircraft, robots, advanced jet engines and complex chemical processes. Existing linear methods are generally adequate to control and/or estimate a nonlinear system over some small part of its range of operation. But to control such a system over its full operating envelope might require tens or even hundreds of linear controllers each valid over a small subregion. Even if each of these controllers perform satisfactorily over its subregion, there is the problem of interpolating these local, linear controllers to obtain a satisfactory global controller. Typically the interpolation is done in an ad hoc fashion and there are no guarantees of performance, robustness or even stability for the resulting controller.

This is the primary reason for the high level of interest and activity in the development of nonlinear controllers and observers. Our research in this area has been guided by two fundamental principles. The first is that any general approach to the design of controllers and observers for nonlinear systems should be based on a proven linear approach. Of course,

there are nonlinear systems for which a particularly nonlinear approach might be successful. But any approach to design for a broad class of nonlinear systems should generalize a linear approach. The reason for this is simple, linear methods generally work well locally. Furthermore, a nonlinear method which does not reduce to a linear method in the small is unlikely to be robust to arbitrary perturbations. If the success of the method depends on a particular relationship between higher order Lie brackets, a small perturbation will generally destroy this relationship and cause the method to fail. For certain classes of nonlinear systems, only particular, nongeneric perturbations are possible. For these classes, nonlinear methods without linear antecedents might be possible. But, for a widely applicable approach, we must focus on nonlinear extensions of linear methods.

There is also a psychological reason for basing nonlinear methods on linear ones. Design engineers will only employ those methods that they are familiar with and in which they have confidence. To be truly confident in a method, the engineer must understand and appreciate the basic approach. This understanding is much more likely to be achieved with a method that is based on and extends familiar linear techniques. Every design method is somewhat idealized and only leads to a template for the actual controller. The template must be modified to meet the particular situation. For example, most design methods ignore the rate and position limits but the final design must take them into account. To make the needed modifications, the design engineer must understand, at least in broad terms, the method that is being used.

The other guiding principle of our research is that ultimately the theory that is being developed must be amenable to computation. Otherwise it cannot be used. During the life of this grant our research has focused on several areas, nonlinear H_∞ control, nonlinear detectability and the solution of PDE's that arise in nonlinear control.

2 Nonlinear H_∞ Control

Over the past decade or more, one of the most active areas of linear research has been the so-called H_∞ control and estimation. The goal has been the development of controllers and estimators with guarantees of performance that are robust in the presence of unknown but gain-bounded perturbations. The original theory was set in the frequency domain and the H_∞ terminology refers to one of the mathematical objects that appeared in its early development.

With the important paper of Doyle, Glover, Khargonekar and Francis [D-G-K-F], the H_∞ terminology became less appropriate as they were able to formulate and solve the problem in state space terms. The optimal H_∞ control problem is difficult to solve directly so the usual approach is to fix a suboptimal level of robust performance denoted by a noise attenuation level called γ and then search for a measurement feedback controller which achieves this level. By iteration on γ, one hopes to converge to the optimal level. The solution of the suboptimal problem separates into two parts, each requiring the solution of a Riccati equation. The first Riccati equation is associated with suboptimal control by state feedback and the second Riccati equation is associated with suboptimal state estimation. There are two variants of this second Riccati equation. The second Riccati equation of Doyle et al. [D-G-K-F] is independent of the first Riccati equation but the two solutions must satisfy a certain

compatibility condition. The second Riccati equation of Tadmor [T] depends on the solutions of the first Riccati equation and does not require a compatibility condition.

The state-space formulation of the linear suboptimal H_∞ control problem has been generalized to nonlinear systems by several groups including van der Schaft [vdS1, vdS2] Başar-Bernhard [B-B], Ball-Helton-Walker [B-H-W] and Isidori-Astolfi [I-A]. James-Baras-Elliot [J-B-E, J-B] have extended the closely related risk sensitive control approach of Whittle to nonlinear systems. These authors were able to give the solution of the nonlinear suboptimal H_∞ control problem by state feedback in term of the solution of a Hamilton-Jacobi-Isaacs partial differential equation, generalizing the first Riccati equation of the linear theory.

But the generalization of the second Riccati equations is more difficult. One approach [I-A] is to linearize the system around zero in the state and control spaces and to use the linearized state estimator as defined by the second Riccati equation of [D-G-K-F]. Another approach [B-B] is based on the theory of differential games and utilizes an auxiliary minimization problem to estimate the state. In a recent paper [K1], we introduced the concept of a conditional storage function and showed how this led to necessary and sufficient conditions for measurement feedback, suboptimal H_∞ control. The state estimator is constucted by solving a second partial differential equation of Hamilton-Jacobi type that is the nonlinear generalization of the second Riccati equation of Tadmor [T].

The first approach leads to a finite dimensional but local compensator while the other two approaches lead to potentially global but infinite dimensional compensators. Each requires the real-time solution of a Hamilton-Jacobi partial differential equation driven by the observations.

3 Detectability of Nonlinear Systems

In a recent article [K2] we surveyed the current state of knowledge regarding stabilizability and detectability of nonlinear system. The literature on nonlinear stabilizability is quite extensive but much less is known about nonlinear detectability. Loosely speaking, a nonlinear system is detectable if admits an observer so that the estimation error converges asymptotically to zero. But it is difficult to make this definition rigorous. More importantly, little is known about the construction of such observers, with the exception of the work of [M],[H], its H_∞ generalization given in [K1] and the work of Baras, Bensoussan and James [B-B-J].

In particular, almost nothing is known about the construction of observers in the critical case, that is, when the local linear approximating system is not detectable in the usual linear sense. (If the local linear approximating system is detectable then an linear observer for it is a local observer for the nonlinear system).

We turn now to a discussion of necessary conditions for detectability analogous to well-known necessary conditions for stabilizability of Brockett. These conditions were developed jointly with M. Zeitz and are valid for both the critical and noncritical systems. For simplicity we restrict our attention to systems with observations but without controls,

$$\dot{x} = f(x) \tag{1}$$

$$y = h(x). \tag{2}$$

Suppose Ω_x is a region of x space where solutions to (1) are unique and where the system is positively invariant, i.e., if $x(0) \in \Omega$ then $\dot{x}(t) \in \Omega$ for all $t \geq 0$. We would like to construct a method for estimating the current state $x(t)$ from the past observations, $\{y(s), 0 \leq s \leq t\}$. Ideally we would like the method to be another dynamical system driven by the observations $y(t)$ whose output is the estimate $\hat{x}(t)$ of $x(t)$, i.e.

$$\dot{z} = g(z, y) \tag{3}$$

$$\hat{x} = k(z, y). \tag{4}$$

Such a system is called an observer and it should have certain properties. At the very least it should be well defined, in other words, there should exist a region Ω_z of z space such that the combined system (1, 2, 3, 4) has unique solutions and is positively invariant on $\Omega_x \times \Omega_z$. In addition, the error

$$e(t) = x(t) - \hat{x}(t) \tag{5}$$

should go to zero as $t \to \infty$ in the sense that there exist a positive definite function $V(e)$ such that $\dot{V}(x, z)$ is negative definitive.

When such an observer exists, we say the nonlinear system (1,2) is detectable.

The following three conditions are necessary for detectability in this sense.

(D_1) If a C^1 system (1,2) admits a C^1 observer with locally asymptotically stable error dynamics, then the linear approximation of (1,2) around any critical point x^0 in Ω_x where $f(x^0) = 0$

$$\dot{x} = \frac{\partial f}{\partial x}(x^0) \tag{6}$$

$$y = \frac{\partial h}{\partial x}(x^0) \tag{7}$$

must have no unobservable, unstable modes.

(D_2) If a C^0 system (1,2) admits a C^0 observer with (locally) asymptotically stable error dynamics and if two (sufficiently close) initial conditions $x^1(0)$ and $x^2(0)$ generate the same output trajectory $y^1(t) = y^2(t)$ then the two state trajectories converge,

$$\left| x^1(t) - x^2(t) \right| \to 0 \tag{8}$$

(D_3) If a C^0 system (1,2) admits a C^0 full order observer with (locally) asymptotically stable error dynamics in the sense of Krener-Zeitz then the mapping

$$x \longmapsto \begin{bmatrix} f(x) \\ h(x) \end{bmatrix} \tag{9}$$

is (locally) 1-1.

Over the past year we have developed a more general theory of observers that is broad enough to include all existing approaches both deterministic and stochastic. We have been able to show that all observers reduce to the solution in a viscosity sense of a partial differential inequality of Hamilton-Jacobi type. We sketch the details.

A more general definition of an observer is a causal mapping

$$\begin{bmatrix} \widehat{x}^0 \\ y(\tau) \end{bmatrix} \longmapsto u(t), \qquad t_0 \le \tau \le t \tag{10}$$

from the initial state estimate $\widehat{x}^0$ and the past observations, $y(s), 0 \le s \le t$, to the current state estimate, $\widehat{x}(t)$. The observver should have the following properties

(1) The estimate $\widehat{x}(t)$ should be a continuous function of t, the initial state estimate $\widehat{x}^0$ and the observation history $y(s), 0 \le s \le t$, the latter equipped with the L^2 or L^∞ norm.

(2) There exists a function $\beta(\nu, t)$ of class K-L such that

$$|x(t) - \widehat{x}(t)| \le \beta(|x(s) - \widehat{x}(s)|, t - s) \tag{11}$$

The justification for condition (1) is more or less obvious, condition (2) is equivalent to the following

(2a) For every $\epsilon > 0$ there exist a $\delta > 0$ such that $|x(s)-\widehat{x}(s)| < \delta$ implies $|x(s)-\widehat{x}(s)| < \epsilon$ for $0 \le s \le t$,

(2b) $|x(t) - \widehat{x}(t)|$ goes to 0 as t goes to ∞,

(2c) if $x(s) = \widehat{x}(s)$ then $x(t) = \widehat{x}(t)$ for all $s \le t$.

Given any such observer there exists a function $Q(x,t)$ with the following properties

(i) $Q(x,t)$ depends causally on $\widehat{x}^0$ and $y(s), 0 \le s \le t$,

(ii) $0 \le Q(x,t) \le \frac{1}{2}|x - \widehat{x}(t)|$,

(iii) $Q(x,t) = 0$ iff $x = \widehat{x}(t)$,

(iv) Along any trajectory $x(t)$, $Q(x(t),t)$ is monotone decreasing to 0,

(v) $Q(x,t)$ is a solution in the viscosity subsolution sense of the partial differential inequality

$$Q_t + Q_x f + \frac{1}{2} Q_x Q_x' - \frac{1}{2}|y - h|^2 \le 0 \tag{12}$$

This result unifies the theory of nonlinear estimation.

4 The PDE's of Nonlinear Control

Many problems of nonlinear control eventually involve the solution of one or more partial differential equations that are fall into one of two classes. The first is the familiar Hamilton-Jacobi equation that appears in optimal control and related problems. The second is the Francis-Byrnes-Isidori PDE and its variants that appears in a host of problems including feedback linearization, input-output linearization, gain-schelduling, nonlinear regulation [K4] and model-matching [K5]. We have developed fast software to solve both these equations by Taylor series expansions. This software is available by ftp from "ftp.utdallas.edu" in the directory "/pub/scadsoftware/nonlineartoolbox". For example the PDE of feedback linearization is solved through cubic terms by the "pc3" routine and the PDE's of optimal regulation [K4] are solved through cubic terms by the "servo3" routine.

But the term by term approach is not always feasible if the solution does not exist or is not sufficiently smooth. A particular case that arises in a variety of problems is the system

of two vector fields in five dimensional space. Generically this system can be approximated by another system with two generators that is free-nilpotent system of degree three using a result from [K3]. The geometry of the extremal (time-optimal) curves for a system with two generators that is free-nilpotent of degree two was completely described by Brockett[Br]. It has become a standard example and/or counterexample in the the theories of nonlinear systems and sub-Riemannian geometry. The geometry of the extremal curves for a system that is free-nilpotent of degree three is much more complicated as it involves elliptic integrals. It is also quite beautiful. The complete synthesis of time-optimal trajectories that project to closed curves was described by Krener and Nikitin [K-N] with support from this grant. These curves have the interesting geometric interpertation as being the solution of a generalized Dido's Problem. They enclose a given area and center of mass by an arc of minimal length.

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